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Properties of Proton in Diquark-Quark Model

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Abstract The properties of the proton has been investigated considering a proton as diquark-quark system. A model for diquark has been suggested in an analogy with the quasi-particle in a crystal lattice. The mass of the diquark obtained in the present model has been found to be in good agreement with other theoretical predictions. The binding energy of proton, compressibility and the excitation energy for the Roper Resonance have been estimated in the context of the proposed model. The results are found to be in agreement with the existing theoretical and experimental findings.

Keywords Diquarks · Binding energy · Compressibility · Roper Resonance

The diquark is strongly correlated quark pair. It is now growing idea that the deeply bound diquarks are the building blocks of the mesons, baryon states and the exotics. The prediction of diquark is as old as the quark model and Gellmann has mentioned the possibility of diquark correlation in his original paper [1]. The regularities in the hadron spetroscopy, parton distribution functions, spin dependent structure function of hadrons etc. hints at the existence of diquark correlation [2–6]. At the experimental level it has been observed that the data for deep inelestic structure function for proton can be fitted in the frame work of the quark-diquark model of proton and it is predicted to be mostly in $u[ud]$ state [7]. A number of works have been done towards the understanding of the structure of diquark. The possibility of forming quark-quark and quark-antiquark system by Instanton Induced Interaction (III) have been developed by Shuryak and Schafer et al. [8, 9]. Betman et al. [10] have investigated formation of bound state of quark-quark or quark-antiquark systems due to instanton induced interaction and predicted that such bound state is formed inside the hadron as a bubble of the size of the instanton radius. In the context of discussing the exotic states of baryon Oka [11, 12] has made a detailed study on the diquark correlation in the context of different models. It has been observed that the diquark picture in the framework of III model

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reproduces mass of the θ^+ baryon within the experimental limits. In QCD both gluon exchange interaction or instanton induced interaction favour spin singlet colour-antisymmetric diquark combination. In the present work we have suggested a model for diquark in which two quarks are supposed to be correlated to form a low energy configuration forming a diquark and behaving like a quasi particle in an analogy with the electron behaving as a quasi particle in the crystal lattice [13]. We have estimated the diquark mass in the framework of the quasi-particle model. The proton is described as diquark-quark ($[ud]_0$–u system) system considering diquark as quasi particle. The binding energy, compressibility of proton have been estimated. We have also estimated the excitation energy of the Roper resonance, the first excited state of the nucleon. The results we obtained are in good agreement with the recent theoretical and experimental findings.

Quasi particle concept is most important in condensed matter physics. Quasi particle is low-lying excitation and corresponds to a single particle whose motion is modified by the interactions within the system. An electron in crystal is subjected to two type of forces namely the effect of crystal field (∇V) and an external force (F) which accelerates the electron [13]. Under the influence of the two forces, an electron in a crystal behaves like a quasi particle whose effective mass m^* reflects the inertia of electrons which are already in a crystal field such that:

$$m^* \frac{dV}{dt} = F. \tag{1}$$

The bare electrons (with normal mass) are affected by the lattice force $-\nabla V$ (where V is the periodic potential) and the external force F so that:

$$m \frac{dV}{dt} = F - \frac{dV}{dx}. \tag{2}$$

Hence the ratio of the normal mass (m) to the effective mass (m^*) can be expressed as:

$$m/m^* = 1 - \frac{1}{F}\left[\frac{\delta \overline{V}}{\delta x}\right]. \tag{3}$$

An elementary particle in vacuum may be suggested to be in a situation exactly the same as an electron in a crystal [13]. We have proposed a similar type of picture for diquark $[ud]_0$ as a quasi particle inside a nucleon. We assume that the diquark is an independent body which is under the influence of two type forces. One is due to the background meson cloud which is represented by potential $\overline{V} = -(4/3)\alpha_s/r$ and it resembles the crystal field on a crystal electron. On the other hand for the external force we have considered an average force $F = -ar$ which is of confinement type. It has been assumed that under the influence of these two type of interaction the diquark is behaving like quasi particle, a low lying excited state and its mass get modified. The ratio of the constituent mass and the effective mass of the diquark can be expressed by using the same formalism as in (3) and is obtained as:

$$\frac{m}{m^*} = 1 + \frac{\alpha_s}{ar^3}. \tag{4}$$

Here m represents the normal constituent mass of the diquark and m^* is the effective mass of the diquark, $\overline{V}$ being the average value of the one gluon exchange type of potential. To calculate the effective mass of the diquark from above expression we need the radius parameter r of the diquark. It may be mentioned that Betman et al. [10] have investigated the possibility of formation of the diquark in the context of the instanton induced interaction model and have considered the instanton spacial distribution function $F(r)$ as $F(r) = \delta(r - \rho_c)$ where ρ_c is the characteristic instanton size. They have shown that the diquark is formed in the form of a bubble of the size of the instanton radius. We have assumed that the 'r' parameter for the diquark is of the order of ρ_c and is taken to be $= 0.4$ fm as stated in [10]. With $\alpha_s = 0.59$ [14] for light hadrons, $r = 0.4$ fm, $m = 0.72$ GeV ($m_u = m_d = 0.36$ GeV) and $a = 0.06\ \text{GeV}^3$ [15], we get the effective mass of the diquark as 272 MeV in the quasi particle model of the diquark. Jaffe et al. [16] have estimated the diquark mass 420 MeV in the context of III whereas KI Model [17] estimated the mass to be 209 MeV. Some other calculations [10, 18, 19] estimated the mass of the diquarks in the range between 600 to 800 MeV.

To estimate the binding energy of proton we have considered $[ud]_0$–u picture of proton where $[ud]_0$ is the diquark of mass 272 MeV as estimated by us in the context of the quasi particle model of diquark. We have obtained the reduced mass (M_R) of the diquark-quark system as 0.155 GeV. We assume that a particle of mass M_R is moving under a harmonic oscillator type of potential $V(r) = ar^2$ with the centre of mass fixed to the centre of the baryonic sphere. It may be mentioned here that Krolikowski [20] has made similar type of one body equilateral triangle approximation to the three (Dirac) particle system to estimate the Hamiltonian of the system. The expression for the Hamiltonian corresponding to the particle of mass M_R moving with a potential $V(r) = ar^2$ may be expressed as:

$$H = -\frac{\hbar^2}{2M_R}\nabla^2 + ar^2. \tag{5}$$

The expectation value of H corresponding to the wave function $\psi(r)$ is:

$$\langle H\rangle = \int \psi^* H \psi d^3r. \tag{6}$$

The lowest upper bound of the ground state energy for a wave function ψ may be recast as:

$$E_0 \le \int \psi^* H \psi d^3r. \tag{7}$$

In the context of the Statistical model [21–23] we have derived an expression for the square of the wave function of a proton which is runs as:

$$\psi(r) = A^{\frac{1}{2}}(r_0^2 - r^2)^{\frac{3}{4}} e^{i\alpha}\theta(r_0 - r). \tag{8}$$

Where r_0 corresponds to the radius parameter of a proton. θ is usual step function. α is a constant phase factor, $A = \frac{8r_0^{-6}}{\pi^2}$ [20]. From (8) we can have

$$\nabla\psi = -\frac{3}{2}A^{\frac{1}{2}}(r_0^2 - r^2)^{-\frac{1}{4}} e^{i\alpha}\theta(r_0 - r). \tag{9}$$

The average kinetic energy $\langle T\rangle$ has been estimated with the above statistical model wave function as an input and is obtained as:

$$\langle T\rangle = 43.49 r_0^{-2}. \tag{10}$$

The expectation value of the potential energy has been estimated as:

$$\langle V\rangle = 0.375 a r_0^2. \tag{11}$$

Hence the total energy is:

$$\langle E\rangle = 43.49 r_0^{-2} + 0.375 a r_0^2. \tag{12}$$

r_0 is typical radius parameter of the proton and can be estimated using the experimental data corresponding to the form factor of the proton. The form factor of proton can be obtained from the expression $F(q^2) = \int e^{iq.r}|\psi(r)|^2 d^3r$. With the above $|\psi(r)|^2$ as an input we have estimated $F(q^2)$ for $q^2 \longrightarrow 0$ as:

$$F(q^2) = 1 - 0.057 r_0^2 q^2. \tag{13}$$

The relation between the proton charge radius and form factor for a proton is:

$$F(q^2) = 1 - \frac{1}{6}\langle r_{ch}^2\rangle q^2. \tag{14}$$

Comparing above expressions for $F(q^2)$ we obtain; $\langle r_{ch}^2\rangle = 0.6 r_0$. With the proton charge radius $\langle r_{ch}^2\rangle^{\frac{1}{2}} = 0.88$ fm [24], the radius parameter of the proton is found to be $r_0 = 1.46$ fm $= 7.33$ GeV. With the input of this radius we have obtained $\langle E\rangle = 1.927$ GeV. It may be mentioned that Lim [25] has solved the three body (trinucleon) problem with the spin dependent internuclear harmonic oscillator potential and obtained the binding energy of the triton as:

$$E = \left(\frac{2\hbar^2}{m}\right)^{1/2}\left\{\left(3\omega V_1 + \frac{3}{2}b_1 V_1\right)^{1/2} + \left(3\omega V_1 - \frac{3}{2}b_1 V_1\right)^{1/2}\right\} - 3\omega V_0 \tag{15}$$

where V_0, V_1, b_1, ω are appropriate constants. Considering the three body system as the three quark system with mass of m as m_q (360 MeV), the quark mass, the binding energy for ground state of proton is obtained as $\langle E\rangle = 1.997$ GeV using the expression (15) (with $\frac{1}{2}Kr^2 = V_2 r^2 = V_1\omega r^2 = ar^2$). It is interesting to observe that the our estimate of the binding energy for proton in the context of the quasi-particle diquark-quark model agrees closely with the Lim's exact theoretical calculation of the ground state energy.

The compressibility of a nucleon is given by the expression:

$$K = r_0^2 \frac{1}{3}(d^2E/dr^2). \tag{16}$$

To get an estimate of the compressibility of nucleon we use the radius parameter of nucleon $= 7.33$ GeV^{-1} as above and get the value as $K_N = 1.61$ GeV.

The Roper excitation energy is given by:

$$\Delta E = \left\{\frac{K_N}{m_q r_0^2}\right\}^{\frac{1}{2}}. \tag{17}$$

We have estimated the Roper excitation energy as 288 MeV in the present work. It may be mentioned that the Roper resonance form factor is smaller than the proton form factor indicates the fact that the Roper being a more diffuse system than proton [26]. Morsch et al. [27, 28] have pointed out that the information on the compressibility of a system can be obtained from the dynamical properties of the size degree of freedom in radial mode. They have investigated $P_{11}(1440)$ in the alpha-photon scattering. With the mean square radius equals to 0.62 fm^2, they have extracted the $K_N = 1.4 \pm 0.3$ GeV. MIT [29] Bag model predicts the value in between 900 to 1200 Mev whereas the constituent quark model yields the value as $\simeq 3$ Gev. Mathieu et al. [30] have estimated the value as 636 MeV in the context of the flux tube model. Hoodbhoy et al. [31] have investigated the pion mediated interaction in a chiral bag model considering nucleon size and compressibility as parameters. They have taken K_N as 2 GeV and 3 GeV with radius parameter as 0.6 fm to 0.8 fm. Meissner et al. [32] estimated the compressibility as 4 GeV.

We have estimated the excitation energy of Roper resonance as 288 MeV with $K_N = 1.61$ GeV as estimated by us. Meissner et al. [32] have estimated the excitation energy to be 390 Mev. However they have mentioned that all the relativistic approaches estimate the Roper resonance excitation energy ranging from 200 MeV to 500 MeV whereas the experiment predicts the value as 500 Mev.

In the present work we have suggested a quasi particle picture of diquark which resembles a quasi particle in a crystal lattice. It would be pertinent to mention here that in the context of investigating the superconducting properties of a hadrons [33, 34], it has been suggested that vacuum is supposed to contain a sea of virtual $q\overline{q}$ pairs condensates somewhat similar to the situation of Cooper pairs in a superconductor. The particle picture description of hadrons may have some relevance to the recently developed idea that diquarks may be the building blocks of hadrons and exotics. We have observed that if a trinucleon system is replaced by three quarks to represent a proton the binding energy obtained from the exact solution of Lim [25] is almost equal to our estimated value obtained from the diquark-quark picture of proton where diquark is represented by a quasi particle. It may be pointed out that analysing the deep inelastic scattering data Fredriksson et al. [7] have observed that the proton can be well described as the $u(ud)_0$ state. In a subsequent analysis [35] they have also shown that a non relativiatic potential model for the proton and neutron predicts the spin 0 diquark structure of rms radius 0.35 fm. The quasi particle model of diquark in the present work yields the diquark mass = 0.272 GeV. The compressibility of the nucleon estimated in the present work lies within the range of recent extracted value [27, 28]. The estimated value of the Roper excitation energy lies in the range of other theoretical estimates.

It should be mentioned here that the most uncertainty lies in the estimation of the radius parameter which is not very well known [27, 28]. In the context of discussing the pentaquark baryons Oka [11, 12] has pointed out that situation is not very clear with understanding of the formation and nature of the diquark but diquarks should be reexamined extensively with the ground state other baryons. The model we have suggested for diquaks reproduces the properties of proton in the existing theoretical and experimental limits and may not be far from reality. However further investigations would be made with the other hadrons and exotics in our future works.

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